Inference at ∗ 1 0
of proof for Lemma fib_wf:

1. n : $\mathbb{N}$
2. $\forall n_1$:$\mathbb{N}$. $(n_1 < n) \Rightarrow (\text{fib}(n_1) \in \mathbb{N})$
$\vdash$ $\text{fib}(n) \in \mathbb{N}$
by PERMUTE{1:n,
2:n,
3:n,
4:n,
5:n,
6:n,
4:n,
7:n,
5:n,
8:n,
9:n,
10:n,
11:n,
12:n,
13:n,
14:n,
15:n,
14:n,
16:n,
17:n,
18:n,
19:n,
20:n,
18:n,
21:n,
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22:n,
23:n,
24:n,
25:n,
26:n,
19:n,
27:n,
20:n,
28:n,
29:n,
30:n,
31:n,

32:n,
33:n,
30:n,
34:n,
31:n,
23:n,
35:n,
36:n,
37:n,
38:n,
32:n,
39:n,
33:n,
24:n,
40:n,
41:n,
37:n,
42:n,
43:n}

1: .....wf..... NILNIL

$\vdash ((n =_0 0) \vee_b (n =_0 1)) \in \mathbb{B}$

2: .....wf..... NILNIL

$\vdash \mathbb{B} \in \text{Type}$

3: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \text{tt}$

$\vdash (((n =_0 0) \vee_b (n =_0 1)) = \text{tt}) \in \mathbb{P}_1$

4: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \text{tt}$

$\vdash (\uparrow((n =_0 0) \vee_b (n =_0 1))) \in \mathbb{P}_1$

5: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \text{tt}$

$\vdash ((n = 0) \vee (n = 1)) \in \mathbb{P}_1$

6: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \text{tt}$

$\vdash ((n =_0 0) \vee_b (n =_0 1)) \in \mathbb{B}$

7: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \text{tt}$

$\vdash ((\uparrow(n =_0 0)) \vee (\uparrow(n =_0 1))) \in \mathbb{P}_1$

8: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \mathrm{tt}$
$\vdash (n =_0 0) \in \mathbb{B}$
9: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \mathrm{tt}$
$\vdash (n =_0 1) \in \mathbb{B}$
10: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \mathrm{tt}$
$\vdash (\uparrow(n =_0 0)) \in \mathbb{P}_1$
11: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \mathrm{tt}$
$\vdash (n = 0) \in \mathbb{P}_1$
12: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \mathrm{tt}$
$\vdash (\uparrow(n =_0 1)) \in \mathbb{P}_1$
13: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \mathrm{tt}$
$\vdash (n = 1) \in \mathbb{P}_1$
14: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \mathrm{tt}$
$\vdash n \in \mathbb{Z}$
15: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \mathrm{tt}$
$\vdash 0 \in \mathbb{Z}$
16: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \mathrm{tt}$
$\vdash 1 \in \mathbb{Z}$
17: .....truecase..... NILNIL

3. $(n = 0) \vee (n = 1)$
$\vdash 1 \in \mathbb{N}$
18: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \mathrm{ff}$
$\vdash (((n =_0 0) \vee_b (n =_0 1)) = \mathrm{ff}) \in \mathbb{P}_1$
19: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \mathrm{ff}$
$\vdash (\uparrow((\neg_b (n =_0 0)) \wedge_b (\neg_b (n =_0 1)))) \in \mathbb{P}_1$
20: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \mathrm{ff}$
$\vdash ((\neg(n = 0)) \ \& \ (\neg(n = 1))) \in \mathbb{P}_1$
21: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \mathrm{ff}$
$\vdash (\uparrow(\neg_b((n =_0 0) \vee_b (n =_0 1)))) \in \mathbb{P}_1$
22: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \mathrm{ff}$
$\vdash ((n =_0 0) \vee_b (n =_0 1)) \in \mathbb{B}$
23: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \mathrm{ff}$
$\vdash (n =_0 0) \in \mathbb{B}$
24: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \mathrm{ff}$
$\vdash (n =_0 1) \in \mathbb{B}$
25: .....antecedent..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \mathrm{ff}$
$\vdash$ True
26: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \mathrm{ff}$
4. $(\uparrow(\neg_b((n =_0 0) \vee_b (n =_0 1)))) = (\uparrow((\neg_b (n =_0 0)) \wedge_b (\neg_b (n =_0 1))))$
$\vdash \mathbb{P}_1 = \mathbb{P}_1$
27: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \mathrm{ff}$
$\vdash ((\uparrow(\neg_b (n =_0 0))) \ \& \ (\uparrow(\neg_b (n =_0 1)))) \in \mathbb{P}_1$
28: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \mathrm{ff}$
$\vdash (\neg_b (n =_0 0)) \in \mathbb{B}$
29: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \mathrm{ff}$
$\vdash (\neg_b (n =_0 1)) \in \mathbb{B}$
30: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \mathrm{ff}$

$\vdash (\uparrow(\neg_b(n =_0 0))) \in \mathbb{P}_1$
31: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \text{ff}$
$\vdash (\neg(n = 0)) \in \mathbb{P}_1$
32: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \text{ff}$
$\vdash (\uparrow(\neg_b(n =_0 1))) \in \mathbb{P}_1$
33: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \text{ff}$
$\vdash (\neg(n = 1)) \in \mathbb{P}_1$
34: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \text{ff}$
$\vdash (\neg(\uparrow(n =_0 0))) \in \mathbb{P}_1$
35: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \text{ff}$
$\vdash (\uparrow(n =_0 0)) \in \mathbb{P}_1$
36: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \text{ff}$
$\vdash (n = 0) \in \mathbb{P}_1$
37: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \text{ff}$
$\vdash n \in \mathbb{Z}$
38: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \text{ff}$
$\vdash 0 \in \mathbb{Z}$
39: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \text{ff}$
$\vdash (\neg(\uparrow(n =_0 1))) \in \mathbb{P}_1$
40: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \text{ff}$
$\vdash (\uparrow(n =_0 1)) \in \mathbb{P}_1$
41: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = \text{ff}$
$\vdash (n = 1) \in \mathbb{P}_1$
42: .....wf..... NILNIL

3. $((n =_0 0) \vee_b (n =_0 1)) = $ ff

$\vdash 1 \in \mathbb{Z}$

43: .....falsecase..... NILNIL

3. $(\neg(n = 0))$ & $(\neg(n = 1))$

$\vdash$ fib(n - 1)+fib(n - 2) $\in \mathbb{N}$

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